



## Course Syllabus: Statistics of Extremes - STAT 380

<b>Division</b>	Computer, Electrical and Mathematical Sciences & Engineering
<b>Course Number</b>	STAT 380
<b>Course Title</b>	Statistics of Extremes
<b>Academic Semester</b>	Spring
<b>Academic Year</b>	2021
<b>Semester Start Date</b>	01/24/2021
<b>Semester End Date</b>	05/11/2021
<b>Class Schedule</b> (Days & Time)	10:15 AM - 11:45 AM   Sun Wed

### Instructor(s)

Name	Email	Phone	Office Location	Office Hours
Raphael Georges Huser	raphael.huser@kaust.edu.sa	+966128080682	4125, 1, Al-Khwarizmi (bldg. 1)	TBA

### Teaching Assistant(s)

Name	Email
TBA	TBA

### Course Information

<b>Comprehensive Course Description</b>	<p><b>Description:</b> This advanced statistics course aims at providing a deep understanding of Extreme-Value Theory results, models, and methods, as well as some experience in the practical application of these tools to real data using the statistical software R. Theoretical and practical aspects will be covered.</p> <p><b>Tentative week-by-week outline:</b></p>
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- **W1: Introduction:** Motivation; Examples; Brief history of the field developments;
- **W1-2: Fluctuations of Sums:** Convergence; Laws of large numbers; Glivenko-Cantelli Theorem; Law of the iterated logarithm; Stable laws; Central Limit Theorem; Domains of attraction;
- **W2: Fluctuations of Maxima:** Max-stability; Extremal Types Theorem (ETT); Generalized Extreme-Value (GEV) distribution; Max-domains of attraction; Von Mises conditions;
- **W3-4: Statistical Analysis of Maxima:** Application of the ETT; Return levels; Moment and likelihood estimation; Bayesian extremes; MCMC; Metropolis-Hastings algorithm;
- **W5-6: Threshold Approaches:** Point processes; Limiting results; Peaks Over Threshold (POT) analysis; Generalized Pareto Distribution (GPD); r-largest statistics method;
- **W7: Modeling of Non-stationarity:** Non-stationarity; Inference and diagnostics; Semi-parametric regression; Quantile regression;
- **W8-9: Temporal Dependence:** Time series; Stationarity;  $D(\text{un})$  and  $D'(\text{un})$  conditions; Clustering and cluster size distribution; Extremal index; Return levels; Declustering;
- **W10-12: Multivariate Extremes:** Componentwise maxima; Standardization; Limit distribution; Exponent measure; Pickands' dependence function; Parametric models; Multivariate point process; Multivariate GPD; Extrapolation; Asymptotic dependence and independence; Dependence measures; Likelihood inference; Composite likelihood Ledford-Tawn model and extensions; Heffernan-Tawn approach;
- **W13-15: Spatial Extremes:** Gaussian processes; Stationarity and isotropy; Correlation functions; Max-stable processes; Limiting results for pointwise maxima; de Haan's representation; Parametric models; Current challenges.

**Course Description from Program Guide**

Recommended Prerequisite: STAT 320, 370. This advanced statistics course aims at providing a rather deep understanding of Extreme-Value Theory results, models, and methods, as well as some experience in the practical application of these tools to real data using the statistical software R. Theoretical and practical aspects will be covered. Topics covered include (a) Univariate Extreme-Value Theory: Extremal-Types Theorem; GEV distribution; return levels; Domains of attraction; Threshold-based methods; GPD distribution; Point process representation; r-largest order statistics approach; Likelihood inference; Modelling of non-stationarity; Dependent time series; Clustering and declustering approaches. (b) Multivariate Extreme-Value Theory: Modelling of componentwise maxima; Spectral representation; Parametric models; Dependence measures; Asymptotic dependence/independence; Threshold methods; Likelihood-based inference. (c) Spatial Extremes: Gaussian processes; correlation functions; Max-stable

	processes and models.
<b>Goals and Objectives</b>	<p><b>Objectives:</b> At the end of this course, the students should be able to:</p> <ul style="list-style-type: none"> <li>• Understand, state, prove all results seen in class and in the weekly assignments.</li> <li>• Apply extreme-value statistics tools in real data applications, fit extreme-value models using various techniques, interpret the results and diagnostic plots, know how to use the statistical software R in this context, and understand and interpret R outputs.</li> <li>• Master Extreme-Value Theory and Statistics in the univariate, multivariate, and spatio-temporal case, when extreme events are defined as block maxima or threshold exceedances.</li> </ul>
<b>Required Knowledge</b>	<b>Prerequisites:</b> STAT 220, 230, 250. Exceptions may be obtained with a good justification by asking the Instructor.
<b>Reference Texts</b>	<ol style="list-style-type: none"> <li>1. Coles (2001) An Introduction to Statistical Modeling of Extreme Values, Springer;</li> <li>2. Embrechts, Kluppelberg and Mikosch (1997) Modelling Extreme Events for Insurance and Finance, Springer;</li> <li>3. Beirlant, Goegebeur, Teugels and Segers (2004) Statistics of Extremes: Theory and Applications, Wiley;</li> <li>4. de Haan and Ferreira (2006) Extreme Value Theory: An Introduction, Springer;</li> <li>5. Resnick (1987) Extreme Values, Regular Variation, and Point Processes, Springer;</li> <li>6. Leadbetter, Lindgren and Rootzen (1983) Extremes and Related Properties of Random Sequences and Processes, Springer;</li> <li>7. Other books: <ul style="list-style-type: none"> <li>• – Galambos (1987) The Asymptotic Theory of Extreme Order Statistics, Krieger</li> <li>• – Gumbel (1958) Statistics of Extremes, Columbia University Press</li> <li>• – Kotz and Nadarajah (2000) Extreme Value Distributions, Imperial College Press</li> <li>• – Finkenstadt and Rootzen (2004) Extreme Values in Finance, Telecommunications and the Environment, CRC</li> <li>• – Reiss and Thomas (2007) Statistical Analysis of Extreme Values, Birkhauser</li> </ul> </li> </ol>
<b>Method of evaluation</b>	<b>30.00%</b> - Course Project(s)

	<p><b>40.00%</b> - Final exam  <b>30.00%</b> - Scientific review article presentation</p>
<b>Nature of the assignments</b>	<p>There are three types of assignments:</p> <ul style="list-style-type: none"> <li>• <b>Weekly homework:</b> This consists of written theoretical exercises, but will NOT be graded. Solutions will be uploaded on a weekly basis.</li> <li>• <b>Paper presentations:</b> During the semester, the students will have to present two scientific papers in class; this will count for 30% of the final grade (15% for each presentation).</li> <li>• <b>Project:</b> Each student will have to do a semester project, where the extreme-value concepts and methods will be applied in practice to a real dataset using the statistical software R. This project, done individually, will be due near the end of the semester. More details will be given as the semester progresses; this will count for 30% of the final grade.</li> </ul>
<b>Course Policies</b>	<ul style="list-style-type: none"> <li>• Class notices and course related information will be posted periodically on the STAT 380 website on Blackboard. Please check regularly for important information. Also, there may be important email communications (like homework hints or a change in the exams date), so it is important to monitor your email account regularly.</li> <li>• <u>Homework assignments</u> will be given throughout the semester. Although homework assignments will not be graded, students are strongly encouraged to do them regularly. Solutions will be regularly posted on the course website.</li> <li>• <u>The required computer software is R</u> (see <a href="http://www.r-project.org">http://www.r-project.org</a>). All examples and datasets will be provided in R code, and posted on the class webpage (on Blackboard). Recommended R packages for extreme-value analysis include <code>evd</code>, <code>evdbayes</code>, <code>isnev</code>, <code>POT</code>, <code>evir</code>, <code>extRemes</code>, <code>fExtremes</code>, <code>HimDimMaxStable</code>, <code>SpatialExtremes</code>, <code>RandomFields</code>, etc. Students are allowed to use other softwares (such as Matlab), although it might be more time-consuming than using R and is at their own risk.</li> <li>• <u>Project:</u> A project, done individually, will be due near the end of the semester. More details will be given as the semester progresses. Late projects will not be accepted.</li> <li>• <u>Final exam:</u> The final exam is <b>closed books and closed notes</b>. However, students are allowed to bring <i>two A4 sheets of notes</i> (4 sides in total), formulae or any other information (no photocopy is allowed).</li> </ul>
<b>Additional Information</b>	

**Tentative Course Schedule**

(Time, topic/emphasis & resources)

Week	Lectures	Topic
1	Sun 01/24/2021 Wed 01/27/2021	<b>Semester starts;</b> <b>Introduction:</b> Motivation; Examples; Brief history of the field developments; Fluctuations of Sums; Convergence; Laws of large numbers; Glivenko-Cantelli Theorem; Law of the iterated logarithm; Stable laws; Central Limit Theorem; Domains of attraction;
2	Sun 01/31/2021 Wed 02/03/2021	<b>Fluctuations of Maxima:</b> Max-stability; Extremal Types Theorem (ETT); Generalized Extreme-Value (GEV) distribution; Max-domains of attraction; Von Mises conditions;
3	Sun 02/07/2021 Wed 02/10/2021	<b>Fluctuations of Maxima:</b> Max-stability; Extremal Types Theorem (ETT); Generalized Extreme-Value (GEV) distribution; Max-domains of attraction; Von Mises conditions; <b>Statistical Analysis of Maxima:</b> Application of the ETT; Return levels; Moment and likelihood estimation; Bayesian extremes; MCMC; Metropolis-Hastings algorithm;
4	Sun 02/14/2021 Wed 02/17/2021	<b>Statistical Analysis of Maxima:</b> Application of the ETT; Return levels; Moment and likelihood estimation; Bayesian extremes; MCMC; Metropolis-Hastings algorithm;
5	Sun 02/21/2021 Wed 02/24/2021	<b>Threshold Approaches:</b> Point processes; Limiting results; Peaks Over Threshold (POT) analysis; Generalized Pareto Distribution (GPD); r-largest statistics method;
6	Sun 02/28/2021 Wed 03/03/2021	<b>Threshold Approaches:</b> Point processes; Limiting results; Peaks Over Threshold (POT) analysis; Generalized Pareto Distribution (GPD); r-largest statistics method;
7	Sun 03/07/2021 Wed 03/10/2021	<b>PAPER PRESENTATIONS 1</b>
8	Sun 03/14/2021 Wed 03/17/2021	<b>MID-SEMESTER BREAK</b> (March 14) <b>Modeling of Non-stationarity:</b> Non-stationarity; Inference and diagnostics; Semi-parametric regression; Quantile regression;
9	Sun 03/21/2021 Wed 03/24/2021	<b>Temporal Dependence:</b> Time series; Stationarity; $D(\text{un})$ and $D'(\text{un})$ conditions; Clustering and cluster size distribution; Extremal index; Return levels; Declustering;
10	Sun 03/28/2021 Wed 03/31/2021	<b>Temporal Dependence:</b> Time series; Stationarity; $D(\text{un})$ and $D'(\text{un})$ conditions; Clustering and cluster size distribution; Extremal

		index; Return levels; Declustering;
11	Sun 04/04/2021 Wed 04/07/2021	<b>Multivariate Extremes:</b> Componentwise maxima; Standardization; Limit distribution; Exponent measure; Pickands' dependence function; Parametric models; Multivariate point process; Multivariate GPD; Extrapolation; Asymptotic dependence and independence; Dependence measures; Likelihood inference; Composite likelihood Ledford-Tawn model and extensions; Heffernan-Tawn approach;
12	Sun 04/11/2021 Wed 04/14/2021	<b>Multivariate Extremes:</b> Componentwise maxima; Standardization; Limit distribution; Exponent measure; Pickands' dependence function; Parametric models; Multivariate point process; Multivariate GPD; Extrapolation; Asymptotic dependence and independence; Dependence measures; Likelihood inference; Composite likelihood Ledford-Tawn model and extensions; Heffernan-Tawn approach;
13	Sun 04/18/2021 Wed 04/21/2021	<b>PAPER PRESENTATIONS 2</b>
14	Sun 04/25/2021 Wed 04/28/2021	<b>Spatial Extremes:</b> Gaussian processes; Stationarity and isotropy; Correlation functions; Max-stable processes; Limiting results for pointwise maxima; de Haan's representation; Parametric models; Current challenges.
15	Sun 05/02/2021 Wed 05/05/2021	<b>Spatial Extremes:</b> Gaussian processes; Stationarity and isotropy; Correlation functions; Max-stable processes; Limiting results for pointwise maxima; de Haan's representation; Parametric models; Current challenges.
16	Sun 05/09/2021	<b>EXAMS WEEK</b>

### Note

The instructor reserves the right to make changes to this syllabus as necessary.